Climate policy coordination in a currency union: Implications for green transition and monetary policy*

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Abstract

In this paper I use scenario analysis to assess the impacts of heterogeneous climate change mitigation policies across euro area member states on the macroeconomy and optimal monetary policy in the euro area. I develop a two-country DSGE model of a currency union extended by a disaggregated energy sector, where fossil and renewable energy are bundled. Each country has a sovereign fiscal authority that levies a carbon tax on fossil energy producers, while monetary policy is conducted by a common central bank. I simulate different green transition scenarios through a linear increase in the carbon tax. My results suggest that if one country delays the transition, it mitigates domestic GDP loss and becomes relatively more competitive. Optimal monetary policy stabilizes union-wide core inflation, even if the transition is not coordinated across countries. The common central bank should therefore "look through" the modest increase in headline inflation resulting from higher energy prices. Finally, the green transition is least harmful in terms of GDP loss if both countries reinvest carbon tax revenues into subsidies to green energy firms.

JEL Codes: E52, F42, Q43

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1 Introduction

In the European Green Deal, the European Union has committed itself to taking measures to ensure the transition to a low-carbon economy with sustainable energy production. This transition is expected to have a significant impact on the macroeconomic dynamics of the euro area, in particular with regard to potential inflationary pressures from higher energy prices, also referred to as "greenflation" (Schnabel (2022)). Following the European Commission's "Fit for 55" package, the EU aims to reduce greenhouse gas emissions by 55% compared to 1990 levels until 2030. This requires an increase in the European carbon price, which is partly harmonized through the EU emission trading system (ETS). However, 60% of EU emissions are generated by so-called "effort sharing" sectors, such as transport and heating, which are subject to national carbon pricing measures of the respective member state, such as carbon taxes. The carbon price from carbon taxes or national ETS systems ranges from 0 to 83 €/tCO_2 in the euro area, implying that there is still considerable heterogeneity in euro area climate policies.[1]

I analyze the macroeconomic implications of an uncoordinated transition to a low-carbon economy in the euro area. The decarbonization of the economy through the increase of carbon prices is found to be effective in terms of emission reduction, but leads to losses in GDP and inflationary pressures (Känzig (2023)). Although the role of monetary policy in climate change mitigation is highly debated, the macroeconomic impact of increasing carbon prices requires central banks to stabilize the economy during the green transition. Konradt and Weder di Mauro (2023) provide empirical evidence from Europe and Canada that carbon taxes might have larger inflationary effects in countries without monetary policy autonomy. Therefore, I use scenario analysis to assess not only the macroeconomic implications of uncoordinated climate policies across member states, but also the optimal response of euro area monetary policy.

I develop a two-country DSGE model of a currency union extended by a disaggregated energy sector. Energy is a bundle of fossil and renewable energy sources. The resulting energy bundle is then used as an additional input factor of production and is part of the final consumption bundle of households. Each country has a sovereign fiscal authority that can levy a carbon tax on fossil energy producers. The revenues from carbon taxation are either rebated to the households via lump-sum transfers or used to subsidize green energy production. Monetary policy is conducted by a common central bank that sets the union-wide nominal interest rate. The model is calibrated to the euro area, specifically the euro area energy sector.

Using this model framework, I simulate different scenarios of a linear increase in the

[1]According to the World Bank carbon pricing dashboard, 11 euro countries have a carbon taxation scheme or national ETS system in place (Austria, Estonia, Finland, France, Germany, Ireland, Latvia, Luxembourg, Portugal, Slovenia and Spain). The respective carbon prices from these initiatives range from approximately 2 €/tCO_2 in Estonia to 83 €/tCO_2 in Finland in 2023.

carbon price to meet the targets of the "Fit for 55" package by 2030. The increase in the carbon price leads to an increase in energy prices, which raises the marginal costs of intermediate goods producers and thus leads to GDP losses and upward pressure on headline inflation. Therefore, my results suggest that both countries have an incentive to delay the green transition. If country F starts to raise the carbon price three years later than country H, production will be shifted to the delaying country during these early years of the transition. As a result, country F improves its competitiveness relative to country H. Both countries experience a reduction in GDP during the transition, but the delay mitigates the GDP loss in country F. However, the delaying country also experiences slightly higher headline inflation once it starts the green transition because the increase in carbon prices must be steeper to meet the targets by 2030.

Optimal monetary policy stabilizes union-wide core inflation during the green transition, even if the transition is delayed in one country. This implies that the central bank should "look through" the increase in energy prices during the green transition and focus on stabilizing core inflation. As a result, there is an increase in headline inflation during the transition, but it turns out to be moderate despite the significant rise in energy prices. Finally, my results suggest that the green transition is least harmful to the economy, when both countries reinvest the revenues from carbon taxation into subsidies for renewable energy firms. With revenue recycling in both countries, there is only a very small reduction in GDP no inflationary pressure during the transition, because the increase in energy prices is significantly mitigated.

The remainder of this paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model and the parameterization. Section 4 discusses the results from different transition scenarios. In Section 5 I examine the robustness of my results to the intoduction of a climate change externality. Finally, Section 6 concludes.

2 Related Literature

The literature addressing the linkage between environmental policy and the macroeconomy using DSGE models has expanded considerably in recent years. It builds on seminal integrated-assessment climate-economy models, such as Nordhaus (2008) and Nordhaus (2017). A growing strand of this literature uses New Keynesian models to study the macroeconomic implications of the transition to a low-carbon economy through increasing carbon prices (Annicchiarico and Di Dio (2015), Diluiso et al. (2021), Benmir and Roman (2020), Känzig (2023) among others). Many papers specifically explore the role of monetary policy, often with regard to the threat of inflationary pressures due to increasing energy prices during the transition.

Ferrari and Nispi Landi (2023) develop a closed-economy New Keynesian model of the euro area that features a polluting and a green production sector to examine the macroeconomic impact of the transition to net-zero emissions by increasing carbon taxes. Their results suggest a reduction in inflation, due to the drop in aggregate demand during the transition. Coenen et al. (2023) extend the ECB's New Area-Wide Model with a disaggregated energy sector to assess the impact of different carbon transition paths on the euro area economy. They model energy as a bundle of green and fossil energy, which is used as an input factor in intermediate goods production and is part of households' final consumption bundle. Their results suggest an increase in headline inflation during the transition due to the increase in energy prices. Similarly, Olovsson and Vestin (2023) find that it is optimal for euro area monetary policy to see through increasing energy prices and focus on stabilizing core inflation, which leads to an increase in headline inflation. However, their results suggest that this increase is modest as long as the carbon tax path is pre-announced. Airaudo et al. (2023) design a small-open economy model with an energy sector and find that an increase in fossil energy prices leads to short-run inflation and persistent output losses. Del Negro et al. (2023) develop a two-sector model to study how the green transition affects the central bank's trade-off between keeping prices stable and closing the output gap. Nakov and Thomas (2023) study Ramsey optimal monetary policy in a model with climate externalities and how it is affected by different environmental policy regimes.

This paper contributes to this literature by examining the macroeconomic implications of the green transition in a two-country setting of a currency union with a common monetary policy. It is therefore also related to literature addressing interactions between monetary and fiscal policy in a monetary union like the euro area.[2]

There is some recent literature on international cooperation in climate change mitigation policies. Ferrari Minesso and Pagliari (2023) develop a three-country DSGE model to identify the optimal mix of monetary and environmental policies to meet the targets from the Paris agreement. Their results suggest that global international cooperation is necessary to reach climate targets and is also optimal from an aggregate welfare perspective. In addition, they find that while optimal monetary policy looks through environmental objectives, it stabilizes the economy when climate policies are implemented. Hassler et al. (2021b) analyze the implications of different carbon tax paths for the transition to a low-carbon economy in a global multi-region model. The authors show that the same tax path has heterogeneous effects across different regions. I contribute to this literature by explicitly focusing on international cooperation within the euro area and the implications for optimal monetary policy.

[2] See Gali and Monacelli (2008), C. J. Erceg and Lindé (2013), Eggertsson et al. (2014), Jarociński and Maćkowiak (2018), Corsetti et al. (2019) among others.

3 The Model

The model is a two-country New Keynesian framework of a monetary union extended by an energy sector. Each country is populated by households, final good producers, intermediate good producers as well as producers of green and fossil energy. The production of fossil energy generates carbon emissions, while green energy production is carbon-neutral. A bundle of green and fossil energy is used for both domestic intermediate goods production and final consumption of domestic households. Both countries have a sovereign fiscal authority, that can levy a carbon tax on fossil energy firms and pay subsidies to green energy firms. Monetary policy is conducted by one common central bank. This section only presents model equations for the home economy, because the specification is symmetric for both countries.

3.1 Households

The representative infinitely-lived household maximizes the following utility function:

$$\max_{c_t,h_t,k_t,i_t,b_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \frac{h_t^{1+\phi}}{1+\phi} \right\}, \tag{1}$$

where c_t represents final consumption, h_t represents hours worked, $\beta \in (0,1)$ is the discount factor and ϕ is the inverse Frisch elasticity. Households maximize lifetime utility subject to their budget constraint, defined as follows in nominal terms:

$$P_t c_t + P_t^I \sum_j i_t^j + B_t = W_t h_t + \sum_j R_t^{k,j} k_{t-1}^j + r_{t-1}^U B_{t-1} + P_t T_t + P_t \Pi_t. \tag{2}$$

Here, P_t is the domestic consumer price index (CPI) and P_t^I is the price of investment $i_t^j, j \in (Y,G,B)$. Investment is allocated across three different sectors: capital goods for intermediate goods production k_t^Y, green energy production k_t^G and fossil energy production k_t^B. Households can invest in one-period bonds B_t, expressed in nominal terms, where $r_{t-1}^U B_{t-1}$ denotes the revenue from holding bonds. $W_t h_t$ is households' labor income, $R_t^{k,j} k_{t-1}^j$ is income from capital services from the respective sector, T_t are lump-sum transfers from the government and Π_t are firm profits.
Following Christiano et al. (2005), households face quadratic adjustment costs in investment, so that investment is smoothed over time. This results in the following capital law of motion for each sector:

$$k_t^j = (1-\delta) k_{t-1}^j + \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t^j}{i_{t-1}^j} - 1 \right)^2 \right] i_t^j, \tag{3}$$

where κ_I denotes the investment adjustment cost parameter.

Maximizing (1) subject to (2) and (3) with respect to c_t and B_t yields the following consumption Euler equation:

$$1 = \beta \mathbb{E}_t \frac{r_t^U}{\pi_{t+1}} \frac{c_t}{c_{t+1}}, \tag{4}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the CPI inflation rate. Combining the consumption Euler equations of both countries yields:

$$x_t = \frac{c_t}{c_t^*}, \tag{5}$$

where $x_t = \frac{P_t^*}{P_t}$ denotes the real exchange rate between country H and country F.
The first-order condition (FOC) with respect to labor h_t is:

$$w_t = h_t^{\phi} c_t, \tag{6}$$

where w_t denotes the real wage.
The FOC with respect to capital $k_t^j, j \in (Y, G, B)$ is:

$$q_t^j = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} \left((1-\delta) q_{t+1}^j + r_{t+1}^{k,j} \right), \tag{7}$$

where q_t^j is the so-called Tobin's q, measuring the marginal value of capital with respect to consumption and $r_t^{k,j}$ is the real return on capital in the respective sector.
Finally, the first order condition with respect to investment i_t^j in each sector is:

$$p_t^I = q_t^j \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t^j}{i_{t-1}^j} - 1 \right)^2 - \kappa_I \left(\frac{i_t^j}{i_{t-1}^j} - 1 \right) \frac{i_t^j}{i_{t-1}^j} \right] + \beta \mathbb{E}_t q_{t+1}^j \left[\frac{c_t}{c_{t+1}} \kappa_I \left(\frac{i_{t+1}^j}{i_t^j} - 1 \right) \left(\frac{i_{t+1}^j}{i_t^j} \right)^2 \right]. \tag{8}$$

To capture the energy consumption of households, final consumption c_t is modeled as a CES bundle of energy (c_t^E) and the manufactured good from final good production (c_t^X), such that

$$c_t = \left(\gamma_c^{\frac{1}{\varrho_c}} (c_t^E)^{\frac{\varrho_c - 1}{\varrho_c}} + (1-\gamma_c)^{\frac{1}{\varrho_c}} (c_t^X)^{\frac{\varrho_c - 1}{\varrho_c}} \right)^{\frac{\varrho_c}{\varrho_c - 1}}. \tag{9}$$

Here, γ_c determines the share of energy in final consumption and ϱ_c is the elasticity of substitution between energy and the manufactured good. The resulting demand equations for energy and the manufactured consumption good are:

$$c_t^E = \gamma_c \left(\frac{P_t^E}{P_t}\right)^{-\varrho_c} c_t, \tag{10}$$

$$c_t^X = (1-\gamma_c) \left(\frac{P_t^X}{P_t}\right)^{-\varrho_c} c_t, \tag{11}$$

where P_t^E and P_t^X are their respective prices. The CPI can therefore be defined such that it captures both goods and energy prices:

$$P_t = \left(\gamma_c (P_t^E)^{1-\varrho_c} + (1-\gamma_c)(P_t^X)^{1-\varrho_c}\right)^{\frac{1}{1-\varrho_c}}. \tag{12}$$

This specification makes it possible to explicitly define a measure for core inflation π_t^X, which, in contrast to headline inflation π_t, excludes fluctuations in energy prices:

$$\pi_t^X = \frac{P_t^X}{P_{t-1}^X} \tag{13}$$

$$= \frac{P_t^X}{P_t} \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-1}^X} \tag{14}$$

$$= \frac{p_t^X}{p_{t-1}^X} \pi_t, \tag{15}$$

where p_t^X denotes the manufactured good price in terms of domestic CPI.
Similarly, energy inflation is defined as follows:

$$\pi_t^E = \frac{p_t^E}{p_{t-1}^E} \pi_t. \tag{16}$$

3.2 International trade

The consumption good excluding energy is a composite of domestically produced and imported goods, such that:

$$c_t^X = \left(\omega_C^{\frac{1}{\eta_C}} (c_{H,t}^X)^{\frac{\eta_C - 1}{\eta_C}} + (1-\omega_C)^{\frac{1}{\eta_C}} (c_{F,t}^X)^{\frac{\eta_C - 1}{\eta_C}}\right)^{\frac{\eta_C}{\eta_C - 1}}, \tag{17}$$

where ω_C determines the home bias in manufactured good consumption and η_C is the elasticity of substitution between home and foreign goods. Note that investment i_t is defined analogously to equation (18) as a bundle of domestically produced investment goods $i_{H,t}$ and imported investment goods $i_{F,t}$:

$$i_t = \left(\omega_I^{\frac{1}{\eta_I}} (i_{H,t})^{\frac{\eta_I - 1}{\eta_I}} + (1-\omega_I)^{\frac{1}{\eta_I}} (i_{F,t})^{\frac{\eta_I - 1}{\eta_I}}\right)^{\frac{\eta_I}{\eta_I - 1}}, \tag{18}$$

where ω_I determines the home bias in investment goods and η_I is the elasticity of substitution between home and foreign investment goods. The resulting demand equations for the domestically produced and imported good are given by:

$$
\begin{aligned}
c_{H,t}^X =& \omega_C \left(\frac{p_{H,t}}{p_t^X}\right)^{-\eta_C} c_t^X, && (19)\\
i_{H,t} =& \omega_I \left(\frac{p_{H,t}}{p_t^X}\right)^{-\eta_I} i_t, && (20)\\
c_{F,t}^X =& (1-\omega_C) \left(\frac{p_{F,t}^X}{p_t^X}\right)^{-\eta_C} c_t^X, && (21)\\
i_{F,t} =& (1-\omega_I) \left(\frac{p_{F,t}^X}{p_t^X}\right)^{-\eta_I} i_t, && (22)
\end{aligned}
$$

where $p_{H,t}$ and $p_{F,t}^X$ are the respective prices in terms of domestic CPI. The price index of manufactured goods relative to domestic CPI is then defined as a composite of the relative price of domestically produced goods and imported goods:

$$p_t^X = \left(\omega_C (p_{H,t})^{1-\eta_C} + (1-\omega_C)(p_{F,t}^X)^{1-\eta_C}\right)^{\frac{1}{1-\eta_C}}. \tag{23}$$

Similarly, the price index of investment goods relative to domestic CPI is defined as follows:

$$p_t^I = \left(\omega_I (p_{H,t})^{1-\eta_I} + (1-\omega_I)(p_{F,t}^I)^{1-\eta_I}\right)^{\frac{1}{1-\eta_I}}. \tag{24}$$

Note that goods prices in country F are defined such that:

$$p_{H,t}^* = \frac{p_{H,t}}{x_t}, \qquad p_{F,t}^* = \frac{p_{F,t}}{x_t}. \tag{25}$$

The net foreign asset position of country H evolves as follows:

$$b_t = \frac{r_t^U}{\pi_{t+1}} b_{t-1} + p_{H,t}\left(c_{H,t}^* + i_{H,t}^*\right) - p_{F,t}\left(c_{F,t} + i_{F,t}\right). \tag{26}$$

Here, the term $p_{H,t}\left(c_{H,t}^* + i_{H,t}^*\right)$ denotes the value of exports of the home economy, while $p_{F,t}\left(c_{F,t} + i_{F,t}\right)$ denotes the value of imports.

Note that I assume that energy cannot be traded across countries for now. Therefore, energy services c_t^E are produced entirely domestically.[3]

[3]This is a simplifying assumption that I plan to relax in a future version of the model.

3.3 Final good firms

The representative domestic final-good firm uses the following CES bundle to produce the final good y_t:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{27}$$

where $y_t(i)$ is an intermediate good produced by intermediate good firm i and ε is the elasticity of substitution between intermediate goods. The profit maximization problem of the final good firm reads as follows:

$$\max_{y_t, \{y_t(i)\}_{i\in[0,1]}} \quad P_{H,t} y_t - \int_0^1 P_{H,t}(i) y_t(i) di \tag{28}$$

$$\text{s.t.} \quad y_t = \left(\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{29}$$

Here, $P_{H,t}(i)$ is the price of the intermediate good produced by firm i in the home country. The problem yields the following intermediate input demand:

$$y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} y_t. \tag{30}$$

3.4 Intermediate good firms

A continuum of intermediate goods $y_t(i)$ is produced by price setting firms that are optimizing under monopolistic competition. The production function of these firms is a CES aggregator in energy and value added from a Cobb-Douglas bundle of capital and labor, following Hassler et al. (2021a):

$$y_t(i) = \left[(1-\gamma_Y)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y(i))^{\alpha} (h_t^Y(i))^{1-\alpha} \right)^{\frac{\varrho_Y - 1}{\varrho_Y}} + (\gamma_Y)^{\frac{1}{\varrho_Y}} \left(e_t^Y(i) \right)^{\frac{\varrho_Y - 1}{\varrho_Y}} \right]^{\frac{\varrho_Y}{\varrho_Y - 1}}, \tag{31}$$

where $e_t^Y(i)$, $k_t^Y(i)$ and $h_t^Y(i)$ respectively is the energy, capital and labor demanded by firm i, α is the capital share in the value added from capital and labor, γ_Y is the energy share in intermediate goods production and ϱ_Y is the elasticity of substitution between energy and the capital-labor bundle.

The firms set prices subject to the demand of the final good firm (30). Firm i pays quadratic price adjustment costs à la Rotemberg (1982), whenever it adjusts its price with respect to the steady-state level of domestic producer price inflation $\pi_{H,t}$:

$$AC_t(i) = \frac{\kappa_P}{2} \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \pi_{H,t} \right)^2 P_{H,t} y_t. \tag{32}$$

The profits of intermediate goods firms, expressed in terms of domestic CPI P_t, are therefore defined as follows:

$$\Pi_t^Y = \frac{P_{H,t}(i)}{P_t} y_t(i) - w_t h_t^Y(i) - r_t^{k,Y} k_{t-1}^Y - p_t^E e_t^Y - \frac{AC_t(i)}{P_t}. \tag{33}$$

Firms set their price $P_{H,t}$ and choose input factors capital, labor and energy to maximize profits subject to their production technology (31) and the demand of the final good firm (30). Since all firms choose the same price in symmetric equilibrium, the index i can be dropped. The maximization problem yields the following first-order conditions:

$$w_t = mc_t \left((1-\gamma_Y) y_t\right)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y)^{\alpha} (h_t^Y)^{1-\alpha}\right)^{\frac{\varrho_Y - 1}{\varrho_Y}} (1-\alpha) \frac{1}{h_t^Y}, \tag{34}$$

$$r_t^{k,Y} = mc_t \left((1-\gamma_Y) y_t\right)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y)^{\alpha} (h_t^Y)^{1-\alpha}\right)^{\frac{\varrho_Y - 1}{\varrho_Y}} \alpha \frac{1}{k_{t-1}^Y}, \tag{35}$$

$$e_t^Y = \left(\frac{p_t^E}{mc_t}\right)^{-\varrho_Y} \gamma_Y y_t, \tag{36}$$

which represent the firms' demand for labor, capital and energy, respectively. Here, mc_t is the Lagrangian multiplier related to marginal costs. Finally, the first-order condition with respect to prices yields the following non-linear Phillips curve:

$$(\pi_{H,t} - \pi_H)\, \pi_{H,t} = \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \frac{p_{H,t+1} y_{t+1}}{p_{H,t} y_t} (\pi_{H,t+1} - \pi_{H,t})\, \pi_{H,t+1}\right] + \frac{\varepsilon}{\kappa_P} \left[\frac{mc_t}{p_{H,t}} - \frac{\varepsilon - 1}{\varepsilon}\right], \tag{37}$$

where domestic producer price inflation is defined as follows:

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t \tag{38}$$

3.5 Energy sector

A representative energy firm combines two different energy sources, green energy e_t^G and brown (fossil) energy e_t^B, to provide energy services to households (c_t^E) and for intermediate goods production (e_t^Y). The energy inputs are bundled using the following CES aggregator:

$$e_t = \left((1-\zeta)^{\frac{1}{\xi}} (e_t^G)^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}} (e_t^B)^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi}{\xi-1}}, \tag{39}$$

where ξ is the elasticity of substitution between green and fossil energy and ζ determines the share of fossil energy in energy production.

Both energy inputs are produced using a Cobb-Douglas bundle of sector-specific capital and labor services k_t^j and h_t^j, $j \in \{B, G\}$. Fossil energy production is assumed to generate carbon emissions m_t, such that:

$$m_t = \vartheta e_t^B, \tag{40}$$

where ϑ determines the carbon content of fossil energy production.

The profit maximization problem of the fossil energy producer is then given by:

$$\max_{k_{t-1}^B, h_t^B} \Pi_t^B = (1-\tau_t) p_t^B \left((k_{t-1}^B)^{\alpha_E} (h_t^B)^{1-\alpha_E} \right) - r_t^{k,B} k_{t-1}^B - w_t h_t^B, \tag{41}$$

where p_t^B is the price of fossil energy and τ_t is a carbon tax that can be levied on fossil energy producers by the domestic fiscal authority.

Similarly, green energy producers maximize profits as follows:

$$\max_{k_{t-1}^G, h_t^G} \Pi_t^G = (1+s_t) p_t^G \left((k_{t-1}^G)^{\alpha_E} (h_t^G)^{1-\alpha_E} \right) - r_t^{k,G} k_{t-1}^G - w_t h_t^G, \tag{42}$$

where p_t^G is the price of green energy and s_t is a subsidy that can be paid by the domestic fiscal authority to support the use of renewable energy sources.

3.6 Monetary and fiscal policy

Both countries have a sovereign fiscal authority that is responsible for implementing environmental policies that foster the transition to a low-carbon economy. I abstract from the existence of public debt and assume the fiscal authorities run a balanced budget at all times. Each government chooses a carbon tax path $\{\tau_t\}_{t=0}^{\infty}$ that is in accordance with the targets set in the European Green Deal. The revenues from carbon taxation can either be used to finance subsidies to renewable energy producers s_t or be rebated to households via lump-sum transfers T_t. The government budget constraint takes the following form:

$$\tau_t p_t^B e_t^B = T_t + s_t p_t^G e_t^G + p_{H,t} g, \tag{43}$$

where government spending g is assumed to be constant. As subsidies s_t are financed by carbon tax revenues, they are defined as follows:

$$s_t = \varsigma \frac{\tau_t p_t^B e_t^B}{p_t^G e_t^G}, \tag{44}$$

where $\varsigma \in [0, 1]$ is the percentage of carbon tax revenues that is reinvested into green subsidies.

Monetary policy is conducted by a common central bank that controls the union-wide nominal interest rate r_t^U. In the benchmark simulations, the central bank follows a standard Taylor rule:

$$\frac{r_t^U}{r^U} = \left(\frac{r_{t-1}^U}{r^U}\right)^{\rho_r} \left[\left(\frac{\pi_t^{X,U}}{\pi^{X,U}}\right)^{\phi_\pi} \left(\frac{gdp_t^U}{gdp_{t-1}^U}\right)^{\phi_y}\right]^{(1-\rho_r)}, \tag{45}$$

where $\pi_t^{X,U}$ describes union-wide core inflation and gdp_t^U is the union-average GDP defined as follows:

$$gdp_t^U = (p_{H,t}y_t)^n \left(p_{F,t}^* y_t^*\right)^{(1-n)}, \tag{46}$$

where $n \in (0,1)$ denotes the size of country H. The central bank is assumed to respond to union-wide core inflation $\pi_t^{X,U}$ rather than headline inflation π_t^U, because well-anchored inflation expectations should allow monetary policy to "look through" temporary inflation fluctuations caused by energy price shocks (C. Erceg et al. (2024)).

3.7 Market clearing

The labor and energy market clear such that:

$$h_t = h_t^Y + h_t^B + h_t^G, \tag{47}$$

$$e_t = c_t^E + e_t^Y. \tag{48}$$

Aggregate investment is defined as follows:

$$i_t = i_t^Y + i_t^B + i_t^G. \tag{49}$$

Aggregating firm profits implies:

$$\Pi_t = \Pi_t^Y + \Pi_t^F + \Pi_t^G. \tag{50}$$

The resource constraint of the home economy is then obtained by plugging the government budget constraint, the profit functions of intermediate goods firms, fossil energy and green energy producers and the definition of the net foreign asset position into the budget constraint of households:

$$p_{H,t}y_t = p_t^X c_t^X + p_t^I i_t + p_{H,t}g + tb_t + \frac{\kappa_p}{2}\left(\pi_{F,t}^X - \pi_F^X\right)^2 p_{H,t}y_t, \tag{51}$$

where

$$tb_t = p_{H,t}\left(c_{H,t}^* + i_{H,t}^*\right) - p_{F,t}\left(c_{F,t} + i_{F,t}\right) \tag{52}$$

represents country H's trade balance, i.e. net exports.
Finally, the bond market clears at:

$$b_t + b_t^* = 0. \tag{53}$$

A full set of equilibrium equations as well as the steady state of the model is listed in appendix A.

3.8 Parameterization

Parameter	Description	Value
β	Quarterly discount factor	0.995
α	Capital share in production	0.3
δ	Capital depreciation rate	0.025
ε	Substitution elasticity intermediate goods	6
ϕ	Inverse Frisch elasticity	2
κ_P	Price adjustment cost parameter	71.2043
κ_I	Investment adjustment cost parameter	10.78
g	Government spending	0.65
ω_C	Home bias consumption	0.81
η_C	Substitution elasticity consumption	2.78
ω_I	Home bias investment	0.74
η_I	Substitution elasticity investment	1.38
n	Country size	0.5
π^U	Steady-state union-wide headline inflation (quarterly)	1.005
$\pi^{X,U}$	Steady-state union-wide core inflation (quarterly)	1.005
π_H^X	Steady-state producer price inflation (quarterly)	1.005
r^U	Steady-state nominal interest rate (quarterly)	1.01
ρ_r	Inertia of Taylor rule	0.93
ϕ_π	Taylor rule core inflation coefficient	2.74
ϕ_y	Taylor rule GDP growth coefficient	0.1
γ_c	Energy share in final consumption	0.07
ϱ_c	Substitution elasticity between energy and manufactured good	0.2
ζ	Green energy share in energy bundle	0.21
γ_Y	Energy share in production	0.15
ϱ_Y	Substitution elasticity between energy and capital-labor bundle	0.2
ξ	Substitution elasticity between fossil and green energy	3
α_E	Share of capital in energy production	0.7
ϑ	Carbon content of fossil energy production	1
ς	Carbon tax revenue recycling percentage	0

Table 1: Parameter values

The model is calibrated to the euro area, at a quarterly frequency. Therefore, I calibrate most economic parameters following the New Area-Wide Model (NAWM-II) in Coenen et al. (2018). Table 1 shows the parameter values. Note that in the baseline version of the model, the two countries are symmetric, implying that the parameter values hold for both countries.

The quarterly discount factor is set to $\beta = 0.995$, which implies an annual steady-state real interest rate of 2%. The steady-state inflation rate is calibrated to match an annual inflation of 2% for both core and headline inflation. The substitution elasticity between intermediate goods is set to $\varepsilon = 6$, which is a standard value in New Keynesian models. The capital share in production is set to $\alpha = 0.3$ and capital depreciates at a rate of $\delta = 2.5\%$ each quarter. Investment adjustment costs are set to $\kappa_i = 10.78$ following Coenen et al. (2018). The inverse Frisch elasticity is set to $\phi = 2$. The price adjustment cost parameter of intermediate good firms is calibrated to $\kappa_P = 71.5603$ following the NAWM-II. Given the substitution elasticity between intermediate goods ε and the discount factor β, this value corresponds to an average price duration of approximately one year. Government spending is calibrated to $g = 0.65$ to match the government spending to GDP ratio of 21.5% given by the NAWM-II. Similarly, the home bias parameters $\omega_C = 0.81$ and $\omega_I = 0.74$ are set to match the import share of 16% in the euro area. The substitution elasticities for consumption and investment are again set to the values estimated in the NAWM-II. The countries are assumed to be equal-sized, which implies $n = 0.5$. The Taylor rule is calibrated such that $\rho_r = 0.93$, $\phi_\pi = 2.74$ and $\phi_y = 0.1$, again following the NAWM-II.

I calibrate the energy-related parameters such that they match the euro area energy sector. According to Eurostat data, the HICP expenditure weight for energy is approximately 10% in the euro area. To replicate this weight in the initial steady state, I set $\gamma_C = 0.07$. Similarly, the share of energy in production is set to $\gamma_Y = 0.15$ to match the ratio of energy expenditure to GDP of 5% in the euro area following Diluiso et al. (2021). The substitution elasticity between energy and the manufactured good in final consumption and the substitution elasticity between energy and the capital-labor bundle in intermediate goods production are set to $\varrho_c = 0.2$ and $\varrho_Y = 0.2$ to match the low elasticities found in Hassler et al. (2021a). The average share of renewable energy in the euro area, according to Eurostat energy statistic, was 21% in 2022. To replicate this share in the initial steady-state of the model, i.e. $\frac{e^G}{e} = 21\%$, I set $\zeta = 0.21$. The substitution elasticity between fossil and green energy is set following Papageorgiou et al. (2017). The authors estimate CES production functions using sectoral data in a panel of 26 countries and find that the elasticity lies between 2 and 3. I set the elasticity to $\xi = 3$, also in line with Acemoglu et al. (2012). The capital share in green and fossil energy production is set to $\alpha_E = 0.7$ to match the capital intensity of energy production following Coenen et al. (2023). Finally, carbon content of fossil energy is assumed to be $\vartheta = 1$ following Hassler et al. (2020).

In the benchmark simulation carbon tax revenues are entirely rebated to households via lump-sum transfers, which implies $\varsigma = 0$.

4 Results

This section presents the results of the model simulations. I simulate a linear increase in the carbon tax from 2023 to 2030, such that carbon emissions are reduced by 30 %. This is the remaining emission reduction needed in the euro area to meet the environmental targets of the "Fit for 55" package. The increase in the carbon tax is permanent, which means that the economy transitions from an initial steady state to a different final one. The model is solved in perfect foresight, implying that the carbon tax increase is pre-announced and all agents have perfect information about the tax path. In the first subsection I analyze the macroeconomic implications of one country delaying the green transition. In section 4.2 I compute the Ramsey optimal monetary policy of the common central bank during the green transition. Section 4.3 assesses the impact of reinvesting carbon tax revenues into subsidies to green energy firms.

4.1 Uncoordinated transition dynamics

Figure 1 shows the macroeconomic implications of a linear carbon tax increase that ensures a 30% emission reduction until 2030. This carbon tax increase also ensures an increase in the renewable energy share $\left(\frac{e_t^G}{e_t}\right)$ to over 42.5%, as targeted in the "Fit-for-55 package". The home country starts to increase the carbon tax immediately, while the foreign country delays it by three years.[4] The foreign country therefore has to increase the tax more steeply to achieve the emission reduction target by 2030. In this scenario I assume that carbon tax revenues are entirely rebated to households via lump-sum transfers.

The carbon tax increase makes fossil energy production more expensive, leading to a significant rise in energy prices in the home country. This leads to a decline in economic activity as the marginal costs of intermediate good producers increase. Home GDP falls gradually, resulting in a permanent decline of about 1.5%. As higher carbon taxes reduce current and expected future real income, households cut back on consumption. However, the fiscal authority rebates revenues from carbon taxation back to households via lump-sum transfers, which mitigates the consumption decline to approximately 0.7%. Aggregate investment in both countries falls by almost 3% during the transition. As fossil energy production becomes more costly due to the carbon tax, the fossil capital stock decreases by 30%, while the green capital stock more than doubles during the transition. This reflects the shift in energy production towards renewable energy sources with higher carbon taxes.

Energy prices in the foreign country do not increase for the first three years of the period considered. This implies that production is shifted to the foreign country due to

[4]Three years is an arbitrary time frame for this simulation. It could also be done with longer or shorter delays.

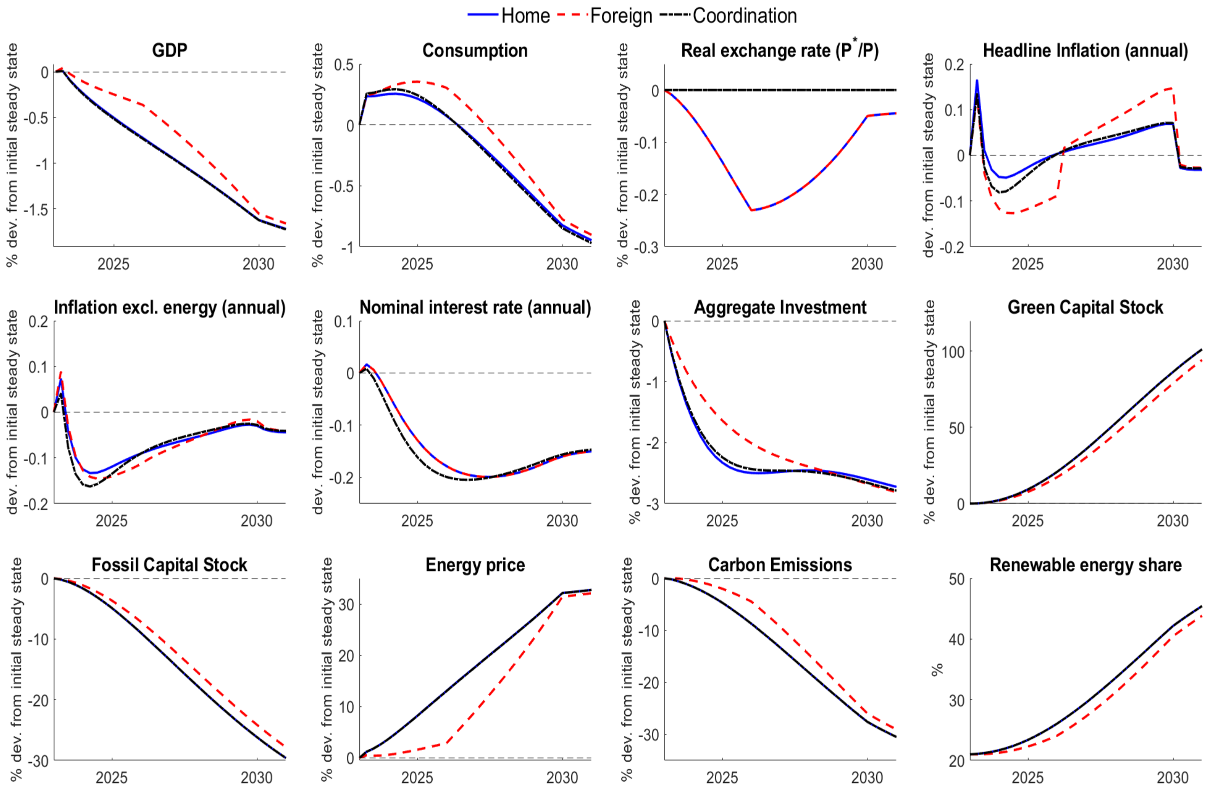

Figure 1: Delayed transition in foreign country

cheaper energy input. As a consequence, the foreign country becomes relatively more competitive, which is reflected in the real exchange rate decline in Figure 1. Once the foreign country starts to increase carbon taxes the real exchange rate reverts to its initial steady state, but the foreign country remains relatively more competitive throughout the transition period. Therefore, GDP loss in the foreign country is mitigated during the transition.

The transition creates a moderate upward pressure on headline inflation, since higher energy prices feed directly into the CPI, as shown in equation (12). Inflation excluding energy rises at first, because intermediate good producers pass on their higher marginal costs from higher energy prices to final goods producers. However, core inflation declines afterwards due to the fall in aggregate demand. The common central bank lowers the nominal interest rate to mitigate the fall in union core inflation. As a result, headline inflation only increases moderately in the home country, with a peak of approximately 17 basis points during the first year of the transition. In the delaying foreign country the first spike in inflation is mitigated and followed by two years of declining inflation. However, foreign headline inflation peaks towards the later years of the transition due to the relatively steep increase in energy prices.

These results imply that both countries have an incentive to delay the transition. The foreign country is better off in terms of GDP loss and competitiveness when it delays the transition than it would be in the case of a coordinated transition. The upward pressure on headline inflation is simply delayed for the foreign country. Core inflation remains relatively stable both in the coordinated and the uncoordinated case.

4.2 Monetary policy

In this section I analyze the Ramsey optimal monetary policy response to different transition scenarios. The central bank acts as a benevolent planner that chooses the optimal trajectory of the nominal interest rate $\{r_t\}$ to maximize union-wide social welfare from a timeless perspective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(U_t)^n (U_t^*)^{1-n} \right], \tag{54}$$

subject to the private sector constraints from the firm's profit maximizing and household's utility maximizing behavior and the path of the carbon tax in both countries $\{\tau_t\}_{t=o}^{\infty}$ and $\{\tau_t^*\}_{t=o}^{\infty}$. The central bank is assumed to commit to the contingent policy rule announced at time 0, which allows dynamic adjustment of the policy instrument to changing economic conditions. Note that I consider a second-best allocation as in Schmitt-Grohé and Uribe (2004) due to inefficiency in the initial steady state, arising from distortive monopolistic competition in intermediate-goods production.

Figure 2 analyzes a coordinated transition and compares the central bank's response under different monetary regimes: (i) Taylor rule as in (45) (benchmark), (ii) Ramsey optimal monetary policy and (iii) Strict core inflation targeting[5].

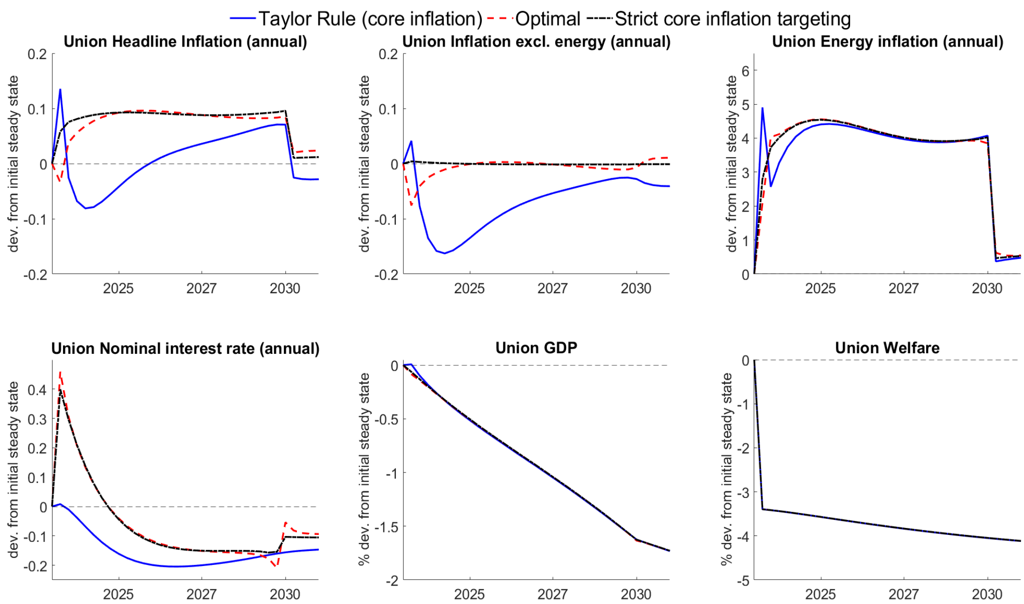

Figure 2: Optimal monetary policy in a coordinated transition scenario

[5]This implies a central bank that puts a very large weight on core inflation stabilization. Specifically, the Taylor rule in (45) is parameterized as follows: $\phi_\pi = 100, \phi_y = 0, \rho_r = 0$.

As discussed in section 4.1, the green transition has a mostly deflationary impact on core inflation and leads to modest inflationary pressures in headline inflation due to the significant increase in energy prices, when the central bank follows the benchmark Taylor rule. When monetary policy is optimal, the central bank aims to stabilize union-wide core inflation. Therefore, the optimal nominal interest rate path is very similar to the path under strict core inflation targeting. At the beginning of the transition, the central bank increases the interest rate to mitigate the increase in core inflation that is caused by rising marginal costs of intermediate goods producers. After that, the central bank lowers the nominal interest rate to mitigate the fall in aggregate demand and thus the fall in core inflation. Therefore, after a small decline during the early years of the transition, union-wide core inflation remains at its steady-state level of 2% annually. This leads to a modest increase in headline inflation during the transition. Annual union-wide headline inflation increases by approximately 8 basis points and stays at this level until the carbon price stops increasing in 2030. Both core and headline inflation are much less volatile under optimal policy and strict core inflation targeting than in the Taylor rule scenario.

Finally, Figure 2 shows that the monetary regime only has very little implications for the evolution of union-wide GDP and welfare during the green transition. The carbon price increase induces union-wide welfare losses of almost 5% relative to the initial no-policy steady state. This significant decline in welfare is barely mitigated by optimal monetary policy. The optimal policy scenario only implies a welfare gain of 0.0001% in consumption equivalent terms during the transition.

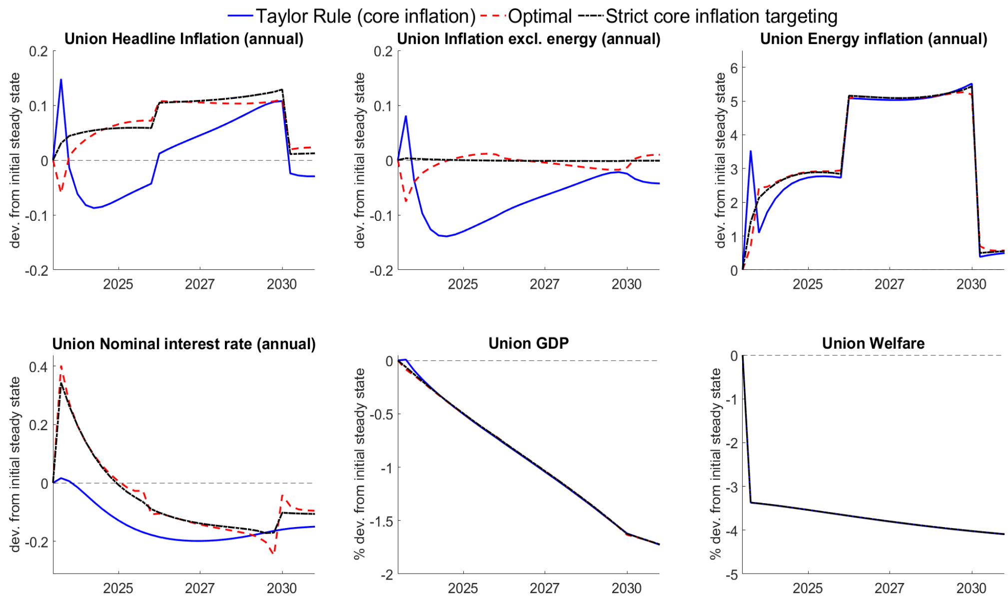

Figure 3: Optimal monetary policy in an uncoordinated transition scenario

The remaining question is whether the optimal monetary policy response changes if the transition is not coordinated between the two countries. Figure 3 again compares the three discussed monetary policy regimes in an uncoordinated transition scenario as in section 4.1. In this scenario the welfare-maximizing central bank also stabilizes union-wide core inflation. This implies a slightly higher lower peak in the nominal interest rate and a slightly lower trough in 2030 than in the coordinated transition scenario, because the carbon price only increases in the home country for the first three years, but then increases more steeply in the foreign country in the subsequent years. However, between the two transition scenarios, the difference in the optimal nominal interest rate path is very small. The average increase in union-wide headline inflation is again approximately 8 basis points, but in the uncoordinated scenario inflationary pressures are very small during the first three years and higher in subsequent years.

To sum up, a welfare-maximizing common central bank chooses the nominal interest path such that it stabilizes union-wide core inflation, even if the green transition is not coordinated between countries. This leads to smaller volatility in both union-wide core and headline inflation.

4.3 Green subsidies

As shown in section 4.1 reducing emissions by increasing carbon taxes entails costs in terms of GDP loss. Therefore, fiscal authorities face a trade-off between achieving climate targets and mitigating losses in GDP during the green transition. Känzig and Konradt (2023) find that this trade-off is less severe, if the government reinvests revenues from carbon pricing initiatives into climate-related purposes, such as green subsidies. Their results suggest that European countries that do not recycle revenues experience a much more significant economic downturn in response to carbon tax increases. Similarly, Konradt and Weder di Mauro (2023) use evidence from European and Canadian carbon tax schemes to show that inflationary pressures caused by carbon taxation are muted in revenue-recycling countries.

Figure 4 illustrates the trade-off between emission reduction and GDP loss. It compares the impact of different levels of carbon taxation on the home country, which fully reinvests carbon tax revenues into subsidies to green energy firms, i.e. $\varsigma = 1$, and on the foreign country, which fully rebates the revenues to households. For the foreign country the trade-off is almost linear: a higher carbon tax leads to lower emissions, at the cost of increasing losses in GDP. Revenue recycling in the home country shifts this trade-off inwards, implying that the same emission reduction can be achieved with less GDP loss. The reason for this effect is twofold. First, revenue recycling makes carbon taxation more effective. The same carbon tax level implies a larger emission reduction in the home country than in the foreign country, because subsidies to renewable energy firms facilitate the transition to non-emitting energy production by making it cheaper. Second, revenue recycling mitigates the negative macroeconomic effects of carbon taxation. As

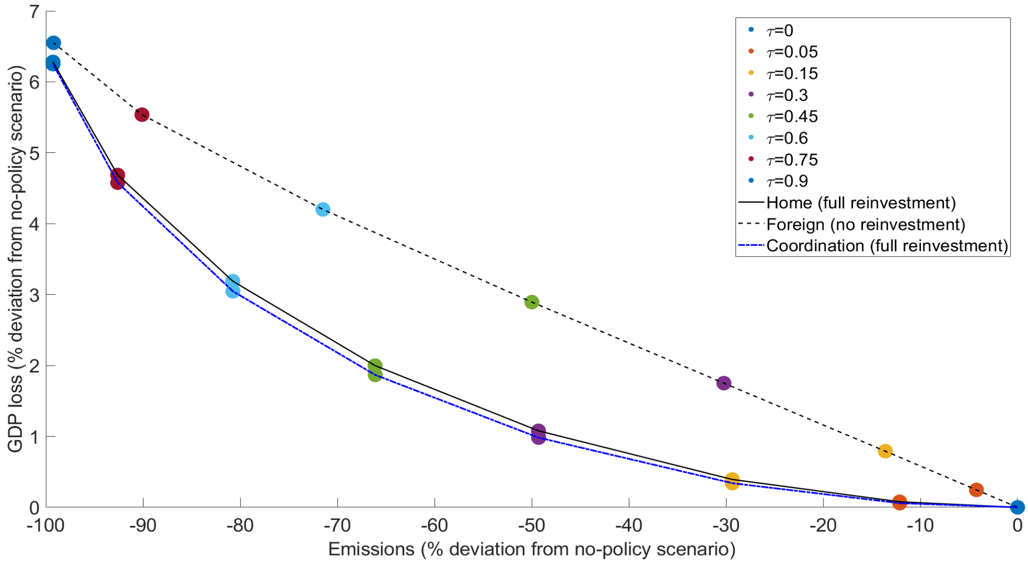

Figure 4: Trade-off between emission reduction and GDP loss

mentioned above, green subsidies lower the input costs of renewable energy producers. Therefore, a higher carbon tax in the home country leads to a less significant increase in energy prices than in the foreign country, because energy production shifts more towards cheaper green energy. The smaller rise in energy prices leads to relatively lower marginal costs for intermediate-good producers and lower energy costs for households, thus mitigating the decline in GDP.

Interestingly, the trade-off between emission reduction and GDP loss shifts even more strongly inwards if both countries fully reinvest carbon tax revenues into green subsidies. As explained above, revenue recycling leads to a less significant increase in marginal costs of intermediate-good producers due to lower energy prices. If both countries recycle revenues, this not only leads to lower prices of domestically produced goods, but also of imported goods. Therefore, coordination with regard to revenue recycling would make both countries better off during the transition. Finally, note that these effects are most pronounced during the transition to a low-carbon economy. Both at the beginning and the end of the transition, the revenues from carbon taxation are very low, implying that there are not many resources available for green subsidies. During the transition, however, carbon tax revenues increase, which makes the effects of revenue recycling much more significant.

To analyze the impact of revenue recycling on the green transition in more detail, Figure 5 compares the transition in reinvesting home country to the one in the foreign country. Both countries impose a linear carbon tax path that leads to a 30% emission reduction by 2030. In line with the results of Känzig and Konradt (2023), the response of GDP to the carbon price increase is very small in the reinvesting home country. As explained above, this is due to the much less significant increase in energy prices. The

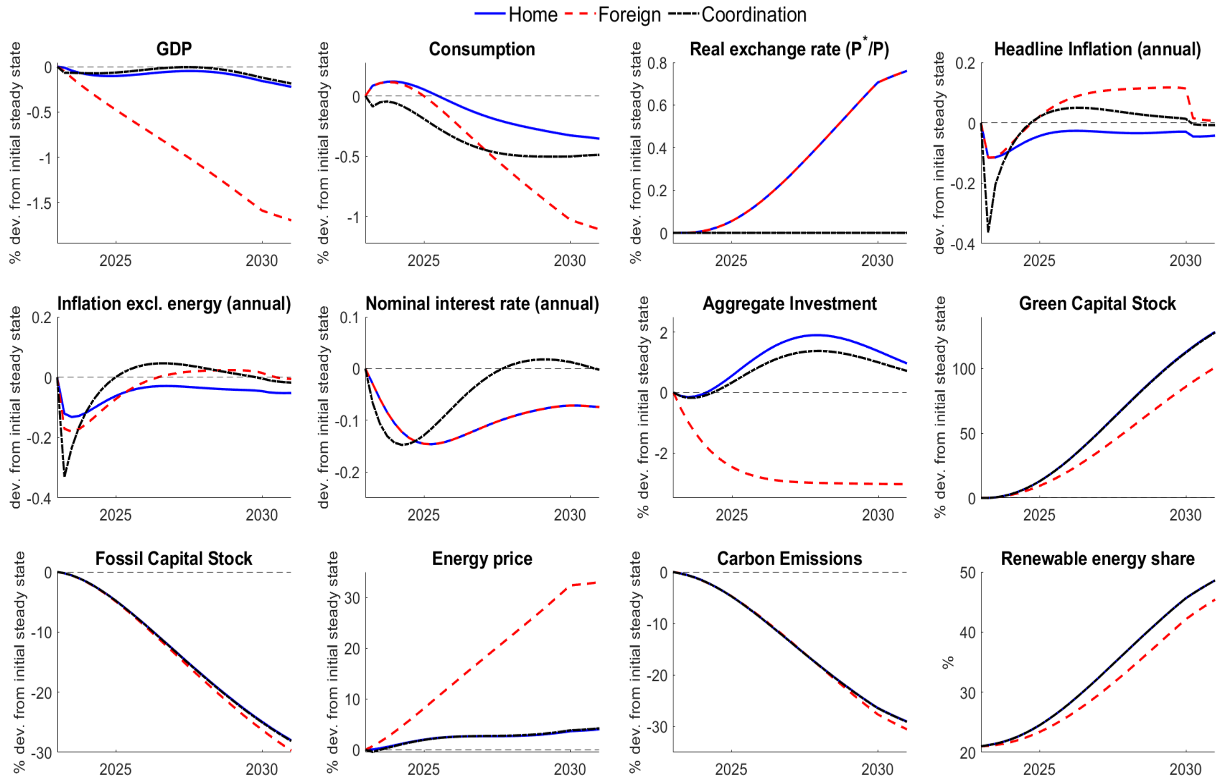

Figure 5: Transition with revenue recycling in home country

home country achieves the same emission reduction as the foreign country, although the imposed carbon tax rate is only half as high. Therefore, intermediate good producers experience a much less significant increase in their marginal costs in the home country. In addition, since green energy production is cheaper in the home country, energy production is shifted towards renewable sources. As a result, aggregate investment increases during the transition when tax revenues are recycled, leading to an even stronger increase in the green capital stock. The decline in consumption is also much less significant in the home country, despite the fact that carbon tax revenues are rebated to households in the foreign country. This is due to less significant decline in current and future expected income in the home country.

Reinvesting tax revenues into green subsidies also makes the home country relatively more competitive during the transition. The real exchange rate gradually and permanently increases by 0.8%. The permanent nature of this gain is due to the lower carbon tax imposed in the home country, which induces permanently lower energy prices than in the foreign country. As predicted by Figure 4, a scenario where both countries fully reinvest carbon tax revenues into green subsidies entails the smallest GDP loss during the transition. Inflationary pressures from the carbon price increase are muted in the home country, in line with the results in Konradt and Weder di Mauro (2023). In contrast, there is a small increase in headline inflation in the foreign country toward the later years of the transition.

5 Robustness

In this section I introduce a climate change externality as in Golosov et al. (2014) into my model to capture negative effects of increasing atmospheric carbon on the economy. The externality creates a two-way interaction between the economy and climate change. In the benchmark model, fossil energy production generates carbon emissions, which feed into the stock of atmospheric carbon. The stock of atmospheric carbon evolves according to the following process:

$$S_t = (1 - \delta_{S,0})S_{t-1} + \delta_{S,1}\left(m_t + m_t^* + m^{row}\right), \tag{55}$$

where $\delta_{S,0}$ is the depreciation rate of carbon dioxide from the atmosphere and $\delta_{S,1}$ is the percentage of carbon emissions that enter the atmosphere. The global stock of atmospheric carbon is fueled by domestic emissions m_t, foreign emissions m_t^* and emissions from the rest of the world m^{row}, which is constant over time, because I assume no climate policy action in the rest of the world.

The extended model now introduces a feedback effect, such that the environmental damage from higher atmospheric carbon reduces total factor productivity.[6]. Following Golosov et al. (2014), total factor productivity in each production sector is then modeled as follows:

$$A_t^i = a_t^i e^{-\psi S_t}, \qquad i \in \{Y, G, B\}, \tag{56}$$

where ψ is the damage parameter that determines the size of the externality and $a_t^i, i \in \{Y, G, B\}$ is the total factor productivity that would prevail in each sector without the environmental externality.[7]

I do not include this specification in the basline version of the model, because the euro area only makes up for less than 10% of global greenhouse-gas emissions. Therefore, climate policy action in the euro area only has a small effect on the evolution of global atmospheric carbon. In addition, the stock of atmospheric carbon decays very slowly over time, implying that the effects of the carbon tax imposed in the model would only materialize long after the year 2030. However, I simulate the same transition scenario as in section 4.1 to check whether the climate externality has an impact on the macroeconomic implications of the carbon tax and whether it changes the optimal policy response of the central bank. I calibrate the climate block of the model as follows: Following Hassler et al. (2020) the damage parameter is set to $\psi = 2.698 * 10^{-5}$, which is the regional-specific

[6]Antoher approach of some environmental DSGE models is to include the pollution externality directly into the utility function of households (see Acemoglu et al. (2012), Benmir et al. (2020), Barrage (2020)). However, Nordhaus (2008) and Heutel (2012) argue that such a modeling choice would be more appropriate for conventional pollutants that directly affect health rather than greenhouse gases.

[7]For simplicity, this is set to $a_t^i = 1$ in each sector.

value they calibrate for the EU. Following Nakov and Thomas (2023), the decay process of atmospheric carbon evolves such that $\delta_{S,0} = 0.00033$ and $\delta_{S,1} = 0.51$.

Figure 6 shows the simulation results of the delayed transition scenario in section 4.1, when the model includes the climate change externality. As mentioned above, the stock of atmospheric carbon decays very slowly over time. During the transition period until 2030, it only declines by approximately 0.01%. Therefore, the macreconomic implications of the transition are barely affected by the environmental externality and the economy evolves as described in section 4.1. The same holds for the optimal monetary policy analysis in section 4.2. Figure 7 shows that with the climate externality, the common central bank still focuses on stabilizing union-wide core inflation, which leads to a modest increase in headline inflation. The optimal policy scenario again only implies a small welfare gain of 0.0005% in consumption equivalent terms during the transition. However, when climate change poses an externality in the model, introducing a carbon tax turns out to be a welfare improvement, because lower fossil energy production will increase total factor productivity over time. Although the climate change externality does not significantly change the transition dynamics, it could be used in future research to examine optimal environmental policy.

6 Conclusion

I simulate different green transition scenarios in a two-country model of a currency union with an energy sector to analyze the impact of heterogeneous carbon taxation in the euro area on the economy and common monetary policy. My results suggest that both countries have an incentive to delay the green transition to mitigate short-term losses in GDP and improve competitiveness, because the carbon price becomes sub-optimally high during the transition. Under optimal monetary policy, the common central bank stabilizes union-wide core inflation. The increase in energy prices therefore leads to a modest and temporary increase in headline inflation. Finally, the trade-off between emission reduction and GDP loss can be mitigated when carbon tax revenues are reinvested in subsidies to renewable energy firms. When both countries engage in revenue recycling, there is only a small decline in GDP and no inflationary pressures during the transition.

This analysis could be extended along a number of dimensions. First, it would be interesting to examine the impact of endogenous technological progress as in Hassler et al. (2021a) and Airaudo et al. (2023) on the transition dynamics. Such directed technical change would facilitate the green transition, as higher carbon prices would create an incentive to allocate more researchers towards making production more energy-efficient. This could aggravate the effects of a heterogeneous climate policies and thus reduce the incentive to delay the transition. Second, in addition to optimal monetary policy, this framework could be used to study optimal environmental policy, i.e. the optimal level of carbon taxation or the optimal tax-subsidy mix. The two-country setting would allow to design environmental policy as a non-cooperative game, as in Minesso and Pagliari (2023), in which each country maximizes its own welfare while taking the policy choice of the other country into account. Finally, in this paper the countries are symmetric in the initial steady state with the only differences arising from heterogeneous climate policies. It could be interesting to make countries heterogeneous in terms of other factors, such as size, energy intensity or initial renewable energy share.

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A Model details

A.1 Equilibrium conditions

The following 82 equations describe the decentralized competitive equilibrium. The overall economy is described by 39 variables related to the home country, $\{c_t, c_t^E, c_t^X, y_t, mc_t, \pi_t, \pi_t^X, \pi_t^E, \pi_{H,t}, i_t, i_t^Y, i_t^G, i_t^B, e_t, e_t^B, e_t^G, e_t^Y, p_t^B, p_t^G, p_t^E, p_t^X, p_t^I, q_t^Y, q_t^G, q_t^B, r_t^{k,Y}, r_t^{k,G}, r_t^{k,B}, w_t, h_t, h_t^B, h_t^G, h_t^Y, k_t^B, k_t^G, k_t^Y, p_{H,t}, p_{F,t}, s_t\}$, 39 variables related to the foreign country, $\{c_t^*, c_t^{E*}, c_t^{X*}, y_t^*, mc_t^*, \pi_t^*, \pi_t^{X*}, \pi_t^{E*}, \pi_{F,t}^*, i_t^*, i_t^{Y*}, i_t^{G*}, i_t^{B*}, e_t^*, e_t^{B*}, e_t^{G*}, e_t^{Y*}, p_t^{B*}, p_t^{G*}, p_t^{E*}, p_t^{X*}, p_t^{I*}, q_t^{Y*}, q_t^{G*}, q_t^{B*}, r_t^{k,Y*}, r_t^{k,G*}, r_t^{k,B*}, w_t^*, h_t^*, h_t^{B*}, h_t^{G*}, h_t^{Y*}, k_t^{B*}, k_t^{G*}, k_t^{Y*}, p_{F,t}^*, p_{H,t}^*, s_t^*\}$, 4 common variables, $\{x_t, r_t^U, \pi_t^{X,U}, gdp_t^U\}$ and an exogenous process for the carbon tax path in each country, $\{\tau_t\}_{t=0}^{\infty}$ and $\{\tau_t^*\}_{t=0}^{\infty}$.

The home country is represented by the following equations:

$$w_t = c_t h_t^{\phi} \tag{A.1}$$

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} \frac{r_t^U}{\pi_{t+1}} \tag{A.2}$$

$$q_t^Y = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,Y} + (1-\delta) q_{t+1}^Y) \tag{A.3}$$

$$q_t^G = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,G} + (1-\delta) q_{t+1}^G) \tag{A.4}$$

$$q_t^B = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,B} + (1-\delta) q_{t+1}^B) \tag{A.5}$$

$$p_t^I = q_t^Y \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t^Y}{i_{t-1}^Y} - 1 \right)^2 - \kappa_I \left(\frac{i_t^Y}{i_{t-1}^Y} - 1 \right) \frac{i_t^Y}{i_{t-1}^Y} \right] + \beta q_{t+1}^Y \left[\frac{c_t}{c_{t+1}} \kappa_I \left(\frac{i_{t+1}^Y}{i_t^Y} - 1 \right) \left(\frac{i_{t+1}^Y}{i_t^Y} \right)^2 \right] \tag{A.6}$$

$$p_t^G = q_t^Y \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t^G}{i_{t-1}^G} - 1 \right)^2 - \kappa_I \left(\frac{i_t^G}{i_{t-1}^G} - 1 \right) \frac{i_t^G}{i_{t-1}^G} \right] + \beta q_{t+1}^G \left[\frac{c_t}{c_{t+1}} \kappa_I \left(\frac{i_{t+1}^G}{i_t^G} - 1 \right) \left(\frac{i_{t+1}^G}{i_t^G} \right)^2 \right] \tag{A.7}$$

$$p_t^I = q_t^B \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t^B}{i_{t-1}^B} - 1 \right)^2 - \kappa_I \left(\frac{i_t^B}{i_{t-1}^B} - 1 \right) \frac{i_t^B}{i_{t-1}^B} \right] + \beta q_{t+1}^B \left[\frac{c_t}{c_{t+1}} \kappa_I \left(\frac{i_{t+1}^B}{i_t^B} - 1 \right) \left(\frac{i_{t+1}^B}{i_t^B} \right)^2 \right] \tag{A.8}$$

$$k_t^Y = (1-\delta) k_{t-1}^Y + \left(1 - \frac{\kappa_I}{2} \left(\frac{i_t^Y}{i_{t-1}^Y} - 1 \right)^2 \right) i_t^Y \tag{A.9}$$

$$k_t^G = (1-\delta) k_{t-1}^G + \left(1 - \frac{\kappa_I}{2} \left(\frac{i_t^G}{i_{t-1}^G} - 1 \right)^2 \right) i_t^G \tag{A.10}$$

$$k_t^B = (1-\delta) k_{t-1}^B + \left(1 - \frac{\kappa_I}{2} \left(\frac{i_t^B}{i_{t-1}^B} - 1 \right)^2 \right) i_t^B \tag{A.11}$$

$$c_t = \left(\gamma_c^{\frac{1}{\varrho_c}} (c_t^E)^{\frac{\varrho_c - 1}{\varrho_c}} + (1-\gamma_c)^{\frac{1}{\varrho_c}} (c_t^X)^{\frac{\varrho_c - 1}{\varrho_c}}\right)^{\frac{\varrho_c}{\varrho_c - 1}} \tag{A.12}$$

$$c_t^E = \gamma_c \left(p_t^E\right)^{-\varrho_c} c_t \tag{A.13}$$

$$c_t^X = (1-\gamma_c) \left(p_t^X\right)^{-\varrho_c} c_t \tag{A.14}$$

$$y_t = \left[(1-\gamma_Y)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y)^{\alpha} (h_t^Y)^{1-\alpha}\right)^{\frac{\varrho_Y - 1}{\varrho_Y}} + (\gamma_Y)^{\frac{1}{\varrho_Y}} (e_t^Y)^{\frac{\varrho_Y - 1}{\varrho_Y}}\right]^{\frac{\varrho_Y}{\varrho_Y - 1}} \tag{A.15}$$

$$w_t = mc_t \left((1-\gamma_Y) y_t\right)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y)^{\alpha} (h_t^Y)^{1-\alpha}\right)^{\frac{\varrho_Y - 1}{\varrho_Y}} (1-\alpha) \frac{1}{h_t^Y} \tag{A.16}$$

$$r_t^{k,Y} = mc_t \left((1-\gamma_Y) y_t\right)^{\frac{1}{\varrho_Y}} \left((k_{t-1}^Y)^{\alpha} (h_t^Y)^{1-\alpha}\right)^{\frac{\varrho_Y - 1}{\varrho_Y}} \alpha \frac{1}{k_{t-1}^Y} \tag{A.17}$$

$$e_t^Y = \left(\frac{p_t^E}{mc_t}\right)^{-\varrho_Y} \gamma_Y y_t \tag{A.18}$$

$$\left(\pi_{H,t} - \pi_H\right) \pi_{H,t} = \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \frac{p_{H,t+1} y_{t+1}}{p_{H,t} y_t} \left(\pi_{H,t+1} - \pi_{H,t}\right) \pi_{H,t+1}\right] + \frac{\varepsilon}{\kappa_P} \left[\frac{mc_t}{p_{H,t}} - \frac{\varepsilon - 1}{\varepsilon}\right] \tag{A.19}$$

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t \tag{A.20}$$

$$\pi_t^X = \frac{p_t^X}{p_{t-1}^X} \pi_t \tag{A.21}$$

$$\pi_t^E = \frac{p_t^E}{p_{t-1}^E} \pi_t \tag{A.22}$$

$$e_t = \left((1-\zeta)^{\frac{1}{\xi}} (e_t^G)^{\frac{\xi - 1}{\xi}} + \zeta^{\frac{1}{\xi}} (e_t^B)^{\frac{\xi - 1}{\xi}}\right)^{\frac{\xi}{\xi - 1}} \tag{A.23}$$

$$e_t^B = \zeta \left(\frac{p_t^B}{p_t^E}\right)^{-\xi} e_t \tag{A.24}$$

$$e_t^G = (1-\zeta) \left(\frac{p_t^G}{p_t^E}\right)^{-\xi} e_t \tag{A.25}$$

$$e_t^B = (k_{t-1}^B)^{\alpha_E} (h_t^B)^{1-\alpha_E} \tag{A.26}$$

$$e_t^G = (k_{t-1}^G)^{\alpha_E} (h_t^G)^{1-\alpha_E} \tag{A.27}$$

$$(1-\alpha_E) p_t^B (1-\tau_t) e_t^B = w_t h_t^B \tag{A.28}$$

$$(1-\alpha_E) p_t^G (1+s_t) e_t^G = w_t h_t^G \tag{A.29}$$

$$\alpha_E p_t^B (1-\tau_t) e_t^B = r_t^{k,B} k_{t-1}^B \tag{A.30}$$

$$\alpha_E p_t^G (1+s_t) e_t^G = r_t^{k,G} k_{t-1}^G \tag{A.31}$$

$$p_t^X = \left(\omega_C (p_{H,t})^{1-\eta_C} + (1-\omega_C)(p_{F,t})^{1-\eta_C}\right)^{\frac{1}{1-\eta_C}} \tag{A.32}$$

$$p_t^I = \left(\omega_I (p_{H,t})^{1-\eta_I} + (1-\omega_I)(p_{F,t})^{1-\eta_I}\right)^{\frac{1}{1-\eta_I}} \tag{A.33}$$

$$h_t = h_t^Y + h_t^B + h_t^G \tag{A.34}$$

$$i_t = i_t^Y + i_t^B + i_t^G \tag{A.35}$$

$$e_t = e_t^Y + c_t^E \tag{A.36}$$

$$s_t = \varsigma \frac{\tau_t p_t^B e_t^B}{p_t^G e_t^G} \tag{A.37}$$

The foreign country is represented by the following equations:

$$w_t^* = c_t^* (h_t^*)^\phi \tag{A.38}$$

$$q_t^{Y*} = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,Y*} + (1-\delta) q_{t+1}^{Y\ *}) \tag{A.39}$$

$$q_t^{G*} = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,G*} + (1-\delta) q_{t+1}^{G\ *}) \tag{A.40}$$

$$q_t^{B*} = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} (r_{t+1}^{k,B*} + (1-\delta) q_{t+1}^{B\ *}) \tag{A.41}$$

$$p_t^{I*} = q_t^{Y*}\left[1 - \frac{\kappa_I}{2}\left(\frac{i_t^{Y*}}{i_{t-1}^{Y\ *}} - 1\right)^2 - \kappa_I \left(\frac{i_t^{Y*}}{i_{t-1}^{Y\ *}} - 1\right)\frac{i_t^{Y*}}{i_{t-1}^{Y\ *}}\right] + \beta q_{t+1}^{Y\ *}\left[\frac{c_t}{c_{t+1}}\kappa_I \left(\frac{i_{t+1}^{Y\ *}}{i_t^{Y*}} - 1\right)\left(\frac{i_{t+1}^{Y\ *}}{i_t^{Y*}}\right)^2\right] \tag{A.42}$$

$$p_t^{I*} = q_t^{G*}\left[1 - \frac{\kappa_I}{2}\left(\frac{i_t^{G*}}{i_{t-1}^{G\ *}} - 1\right)^2 - \kappa_I \left(\frac{i_t^{G*}}{i_{t-1}^{G\ *}} - 1\right)\frac{i_t^{G*}}{i_{t-1}^{G\ *}}\right] + \beta q_{t+1}^{G\ *}\left[\frac{c_t}{c_{t+1}}\kappa_I \left(\frac{i_{t+1}^{G\ *}}{i_t^{G*}} - 1\right)\left(\frac{i_{t+1}^{G\ *}}{i_t^{G*}}\right)^2\right] \tag{A.43}$$

$$p_t^{I*} = q_t^{B*}\left[1 - \frac{\kappa_I}{2}\left(\frac{i_t^{B*}}{i_{t-1}^{B\ *}} - 1\right)^2 - \kappa_I \left(\frac{i_t^{B*}}{i_{t-1}^{B\ *}} - 1\right)\frac{i_t^{B*}}{i_{t-1}^{B\ *}}\right] + \beta q_{t+1}^{B\ *}\left[\frac{c_t}{c_{t+1}}\kappa_I \left(\frac{i_{t+1}^{B\ *}}{i_t^{B*}} - 1\right)\left(\frac{i_{t+1}^{B\ *}}{i_t^{B*}}\right)^2\right] \tag{A.44}$$

$$k_t^{Y*} = (1-\delta) k_{t-1}^{Y\ *} + \left(1 - \frac{\kappa_I}{2}\left(\frac{i_t^{Y*}}{i_{t-1}^{Y\ *}} - 1\right)^2\right) i_t^{Y*} \tag{A.45}$$

$$k_t^{G*} = (1-\delta) k_{t-1}^{G\ *} + \left(1 - \frac{\kappa_I}{2}\left(\frac{i_t^{G*}}{i_{t-1}^{G\ *}} - 1\right)^2\right) i_t^{G*} \tag{A.46}$$

$$k_t^{B*} = (1-\delta) k_{t-1}^{B\ *} + \left(1 - \frac{\kappa_I}{2}\left(\frac{i_t^{B*}}{i_{t-1}^{B\ *}} - 1\right)^2\right) i_t^{B*} \tag{A.47}$$

$$c_t^* = \left(\gamma_c^{\frac{1}{\varrho_c}} (c_t^{E*})^{\frac{\varrho_c - 1}{\varrho_c}} + (1-\gamma_c)^{\frac{1}{\varrho_c}} (c_t^{X*})^{\frac{\varrho_c - 1}{\varrho_c}}\right)^{\frac{\varrho_c}{\varrho_c - 1}} \tag{A.48}$$

$$c_t^{E*} = \gamma_c \left(p_t^{E*}\right)^{-\varrho_c} c_t^* \tag{A.49}$$

$$c_t^{X*} = (1-\gamma_c) \left(p_t^{X*}\right)^{-\varrho_c} c_t^* \tag{A.50}$$

$$y_t^* = \left[(1-\gamma_Y)^{\frac{1}{\varrho_Y}} \left(({k_{t-1}^Y}^*)^{\alpha}(h_t^{Y*})^{1-\alpha}\right)^{\frac{\varrho_Y-1}{\varrho_Y}} + (\gamma_Y)^{\frac{1}{\varrho_Y}} \left(e_t^{Y*}\right)^{\frac{\varrho_Y-1}{\varrho_Y}}\right]^{\frac{\varrho_Y}{\varrho_Y-1}} \tag{A.51}$$

$$w_t^* = mc_t^* \left((1-\gamma_Y) y_t^*\right)^{\frac{1}{\varrho_Y}} \left(({k_{t-1}^Y}^*)^{\alpha}(h_t^{Y*})^{1-\alpha}\right)^{\frac{\varrho_Y-1}{\varrho_Y}} (1-\alpha)\frac{1}{h_t^{Y*}} \tag{A.52}$$

$$r_t^{k,Y*} = mc_t^* \left((1-\gamma_Y) y_t^*\right)^{\frac{1}{\varrho_Y}} \left(({k_{t-1}^Y}^*)^{\alpha}(h_t^{Y*})^{1-\alpha}\right)^{\frac{\varrho_Y-1}{\varrho_Y}} \alpha\frac{1}{{k_{t-1}^Y}^*} \tag{A.53}$$

$$e_t^{Y*} = \left(\frac{p_t^{E*}}{mc_t^*}\right)^{-\varrho_Y} \gamma_Y y_t^* \tag{A.54}$$

$$\left(\pi_{F,t}^* - \pi_F^*\right)\pi_{F,t}^* = \beta \mathbb{E}_t \left[\frac{c_t^*}{c_{t+1}^*}\frac{p_{F,t+1}^* y_{t+1}^*}{p_{F,t}^* y_t^*}\left(\pi_{F,t+1}^* - \pi_{F,t}^*\right)\pi_{F,t+1}^X\right] + \frac{\varepsilon}{\kappa_P}\left[\frac{mc_t^*}{p_{F,t}^*} - \frac{\varepsilon-1}{\varepsilon}\right] \tag{A.55}$$

$$\pi_{F,t}^* = \frac{p_{F,t}^*}{p_{F,t-1}^*}\pi_t^* \tag{A.56}$$

$$\pi_t^{X*} = \frac{p_t^{X*}}{{p_{t-1}^X}^*}\pi_t^* \tag{A.57}$$

$$\pi_t^{E*} = \frac{p_t^{E*}}{{p_{t-1}^E}^*}\pi_t^* \tag{A.58}$$

$$e_t^* = \left((1-\zeta)^{\frac{1}{\xi}}(e_t^{G*})^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}}(e_t^{B*})^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi}{\xi-1}} \tag{A.59}$$

$$e_t^{B*} = \zeta\left(\frac{p_t^{B*}}{p_t^{E*}}\right)^{-\xi} e_t^* \tag{A.60}$$

$$e_t^{G*} = (1-\zeta)\left(\frac{p_t^{G*}}{p_t^{E*}}\right)^{-\xi} e_t^* \tag{A.61}$$

$$e_t^{B*} = ({k_{t-1}^B}^*)^{\alpha_E}(h_t^{B*})^{1-\alpha_E} \tag{A.62}$$

$$e_t^{G*} = ({k_{t-1}^G}^*)^{\alpha_E}(h_t^{G*})^{1-\alpha_E} \tag{A.63}$$

$$(1-\alpha_E)p_t^{B*}(1-\tau_t^*)e_t^{B*} = w_t^* h_t^{B*} \tag{A.64}$$

$$(1-\alpha_E)p_t^{G*}(1+s_t^*)e_t^{G*} = w_t^* h_t^{G*} \tag{A.65}$$

$$\alpha_E p_t^{B*}(1-\tau_t^*)e_t^{B*} = r_t^{k,B*}{k_{t-1}^B}^* \tag{A.66}$$

$$\alpha_E p_t^{G*}(1+s_t^*)e_t^{G*} = r_t^{k,G*}{k_{t-1}^G}^* \tag{A.67}$$

$$p_t^{X*} = \left(\omega_C (p_{F,t}^*)^{1-\eta_C} + (1-\omega_C)(p_{H,t}^*)^{1-\eta_C}\right)^{\frac{1}{1-\eta_C}} \tag{A.68}$$

$$p_t^{I*} = \left(\omega_I (p_{F,t}^*)^{1-\eta_I} + (1-\omega_I)(p_{H,t}^*)^{1-\eta_I}\right)^{\frac{1}{1-\eta_I}} \tag{A.69}$$

$$h_t^* = h_t^{Y*} + h_t^{B*} + h_t^{G*} \tag{A.70}$$

$$k_t^* = k_t^{Y*} + k_t^{B*} + k_t^{G*} \tag{A.71}$$

$$e_t^* = e_t^{Y*} + c_t^{E*} \tag{A.72}$$

$$s_t^* = \varsigma \frac{\tau_t^* p_t^{B*} e_t^{B*}}{p_t^{G*} e_t^{G*}} \tag{A.73}$$

The remaining 9 equations are common equations that hold for both countries:

$$x_t = \frac{c_t}{c_t^*} \tag{A.74}$$

$$\frac{x_t}{x_{t-1}} = \frac{\pi_t^*}{\pi_t} \tag{A.75}$$

$$p_{H,t} = x_t p_{H,t}^* \tag{A.76}$$

$$p_{F,t} = x_t p_{F,t}^* \tag{A.77}$$

$$\frac{r_t^U}{r^U} = \left(\frac{r_{t-1}^U}{r^U}\right)^{\rho_r} \left[\left(\frac{\pi_t^{X,U}}{\pi^{X,U}}\right)^{\phi_\pi} \left(\frac{gdp_t^U}{gdp_{t-1}^U}\right)^{\phi_y}\right]^{(1-\rho_r)} \tag{A.78}$$

$$\pi_t^{X,U} = \left(\pi_t^X\right)^n \left((\pi_t^X)^*\right)^{(1-n)} \tag{A.79}$$

$$gdp_t^U = (p_{H,t} y_t)^n \left(p_{F,t}^{X*} y_t^*\right)^{(1-n)} \tag{A.80}$$

$$\begin{aligned} y_t = {} & \omega_C \left(\frac{p_{H,t}}{p_t^X}\right)^{-\eta_C} c_t^X + \omega_I \left(\frac{p_{H,t}}{p_t^I}\right)^{-\eta_I} i_t + (1-\omega_C)\left(\frac{p_{H,t}^*}{p_t^{X*}}\right)^{-\eta_C} c_t^{X*} \\ & + (1-\omega_I)\left(\frac{p_{H,t}^*}{p_t^{I*}}\right)^{-\eta_I} i_t^* + g + \frac{\kappa_P}{2}\left(\pi_{H,t} - \pi_H\right)^2 y_t \end{aligned} \tag{A.81}$$

$$\begin{aligned} y_t^* = {} & \omega_C \left(\frac{p_{F,t}^*}{p_t^{X*}}\right)^{-\eta_C} c_t^{X*} + \omega_I \left(\frac{p_{F,t}^*}{p_t^{I*}}\right)^{-\eta_I} i_t^* + (1-\omega_C)\left(\frac{p_{F,t}}{p_t^X}\right)^{-\eta_C} c_t^X \\ & + (1-\omega_I)\left(\frac{p_{F,t}}{p_t^I}\right)^{-\eta_I} i_t + g^* + \frac{\kappa_P}{2}\left(\pi_{F,t}^* - \pi_F\right)^2 y_t^* \end{aligned} \tag{A.82}$$

A.2 Steady state

The full model consists of 70 equations and 70 endogenous variables. In steady state the nonlinear model can be simplified to a system of eight equations and eight variables $\{y, p^B, p^E, e, p_H, y^*, p^{B^*}, p^{E^*}\}$. All remaining model variables can be expressed as a function of these eight varibales, which yields the following formulation of the model:

$$p^X = \left(\frac{1 - \gamma_c (p^E)^{1-\varrho_c}}{1 - \gamma_c} \right)^{\frac{1}{1-\varrho_c}}$$

$$p^{X*} = \left(\frac{1 - \gamma_c (p^{E*})^{1-\varrho_c}}{1 - \gamma_c} \right)^{\frac{1}{1-\varrho_c}}$$

$$p_F = \left(\frac{(p^X)^{1-\eta_C} - \omega_C p_H^{1-\eta_C}}{1 - \omega_C} \right)^{\frac{1}{1-\eta_C}}$$

$$x = \frac{1}{p^{X*}} \left((1 - \omega_C) p_H^{1-\eta_C} + \omega_C p_F^{1-\eta_C} \right)^{\frac{1}{1-\eta_C}}$$

$$p_F^* = \frac{p_F}{x}$$

$$p_H^* = \left(\frac{(p^{X*})^{1-\eta_C} - \omega_C (p_F^*)^{1-\eta_C}}{1 - \omega_C} \right)^{\frac{1}{1-\eta_C}}$$

$$p^I = \left(\omega_I (p_H)^{1-\eta_I} + (1 - \omega_I)(p_F)^{1-\eta_I} \right)^{\frac{1}{1-\eta_I}}$$

$$p^{I*} = \left(\omega_I (p_F^*)^{1-\eta_I} + (1 - \omega_I)(p_H^*)^{1-\eta_I} \right)^{\frac{1}{1-\eta_I}}$$

$$mc = p_H \frac{\varepsilon - 1}{\varepsilon}$$

$$e^Y = mc^{-\varrho_Y} \gamma_Y (p^E)^{-\varrho_Y} y$$

$$r^{k,Y} = p^I \left(\frac{1}{\beta} - (1 - \delta) \right)$$

$$r^{k,G} = p^I \left(\frac{1}{\beta} - (1 - \delta) \right)$$

$$r^{k,B} = p^I \left(\frac{1}{\beta} - (1 - \delta) \right)$$

The following equation depicts a help term $va = (k^Y)^\alpha (h^Y)^{1-\alpha}$ to simplify the formulation of other variables:

$$va = \left(\frac{y^{\frac{-\varrho_Y - 1}{-\varrho_Y}} - \gamma_Y^{\frac{1}{-\varrho_Y}} (e^Y)^{\frac{-\varrho_Y - 1}{-\varrho_Y}}}{(1-\gamma_Y)^{\frac{1}{-\varrho_Y}}} \right)^{\frac{\varrho_Y}{\varrho_Y - 1}}$$

$$k^Y = \frac{mc}{r^{k,Y}} \alpha (y(1-\gamma_Y))^{\frac{1}{\varrho_Y}} va^{\frac{\varrho_Y - 1}{-\varrho_Y}}$$

$$h^Y = \left(\frac{va}{(k^Y)^\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$w = \frac{mc}{h^Y} (1-\alpha)((1-\gamma_Y)y)^{\frac{1}{-\varrho_Y}} va^{\frac{-\varrho_Y - 1}{-\varrho_Y}}$$

$$p^G = \left(\frac{1}{1-\zeta} (p^E)^{1-\xi} - \zeta (p^B)^{1-\xi} \right)^{\frac{1}{1-\xi}}$$

$$e^B = \zeta \left(\frac{p^B}{p^E} \right)^{-\xi} e$$

$$e^G = (1-\zeta) \left(\frac{p^G}{p^E} \right)^{-\xi} e$$

$$s = \varsigma \frac{\tau p^B e^B}{p^G e^G}$$

$$k^G = \alpha_E p^G (1+s) e^G \frac{1}{r^{k,G}}$$

$$k^B = \alpha_E p^B (1-\tau) e^B \frac{1}{r^{k,B}}$$

$$h^G = \left(\frac{e^G}{(k^G)^{\alpha_E}} \right)^{\frac{1}{1-\alpha_E}}$$

$$h^B = \left(\frac{e^B}{(k^B)^{\alpha_E}} \right)^{\frac{1}{1-\alpha_E}}$$

$$i^Y = \delta k^Y$$

$$i^G = \delta k^G$$

$$i^B = \delta k^B$$

$$i = i^G + i^B + i^Y$$

$$h = h^G + h^B + h^Y$$

$$c = \frac{w}{h^\phi}$$

$$c^E = \gamma_c (p^E)^{-\varrho_c} c$$

$$c^X = (1-\gamma_c)(p^X)^{-\varrho_c} c$$

$$c^* = \frac{c}{x}$$

$$c^{E*} = \gamma_c (p^{E*})^{-\varrho_c} c^*$$

$$c^{X*} = (1-\gamma_c)(p^{X*})^{-\varrho_c} c^*$$

$$mc^* = p_F^* \frac{\varepsilon - 1}{\varepsilon}$$

$$r^{k,Y*} = p^{I*} \left(\frac{1}{\beta} - (1-\delta) \right)$$

$$r^{k,G*} = p^{I*} \left(\frac{1}{\beta} - (1-\delta) \right)$$

$$r^{k,B*} = p^{I*} \left(\frac{1}{\beta} - (1-\delta) \right)$$

$$e^{Y*} = (mc^*)^{-\varrho_Y} \gamma_Y (p^{E*})^{-\varrho_Y} y^*$$

$$e^* = e^{Y*} + c^{E*}$$

$$p^{G*} = \left(\frac{1}{1-\zeta} (p^{E*})^{1-\xi} - \zeta (p^{B*})^{1-\xi} \right)^{\frac{1}{1-\xi}}$$

$$e^{B*} = \zeta \left(\frac{p^{B*}}{p^{E*}} \right)^{-\xi} e^*$$

$$e^{G*} = (1-\zeta) \left(\frac{p^{G*}}{p^{E*}} \right)^{-\xi} e^*$$

$$s^* = \varsigma \frac{\tau p^{B*} e^{B*}}{p^{G*} e^{G}{*}}$$

$$k^{G*} = \alpha_E p^{G*} (1+s^*) e^{G*} \frac{1}{r^{k,G*}}$$

$$k^{B*} = \alpha_E p^{B*} (1-\tau^*) e^{B*} \frac{1}{r^{k,B*}}$$

$$h^{G*} = \left(\frac{e^{G*}}{(k^{G*})^{\alpha_E}} \right)^{\frac{1}{1-\alpha_E}}$$

$$va^* = \left(\frac{(y^*)^{\frac{-\varrho_Y - 1}{-\varrho_Y}} - \gamma_Y^{\frac{1}{-\varrho_Y}} (e^{Y*})^{\frac{-\varrho_Y - 1}{-\varrho_Y}}}{(1-\gamma_Y)^{\frac{1}{-\varrho_Y}}} \right)^{\frac{\varrho_Y}{\varrho_Y - 1}}$$

$$w^* = (1-\alpha_E) p^{G*} (1+s^*) \frac{e^{G*}}{h^{G*}}$$

$$k^{Y*} = \frac{mc^*}{r^{k,Y*}} \alpha (y^*(1-\gamma_Y))^{\frac{1}{\varrho_Y}} (va^*)^{\frac{\varrho_Y - 1}{-\varrho_Y}}$$

$$h^{Y*} = \frac{mc^*}{w^*}(1-\alpha)(y^*(1-\gamma_Y))^{\frac{1}{\varrho_Y}}(va^*)^{\frac{\varrho_Y - 1}{-\varrho_Y}}$$

$$i^{Y*} = \delta k^{Y*}$$

$$i^{G*} = \delta k^{G*}$$

$$i^{B*} = \delta k^{B*}$$

$$i^* = i^{G*} + i^{B*} + i^{Y*}$$

$$h^* = h^{G*} + h^{B*} + h^{Y*}$$

This model is then solved using the MATLAB routine `fsolve`. Starting from an initial guess, the solver adjusts the values for $\{y, p^B, p^E, e, p_H, y^*, p^{B^*}, p^{E^*}\}$ in each iteration until the following 8 equations hold with arbitrary precision:

$$w = (1-\alpha_E)p^B(1-\tau)\frac{e^B}{h^B}$$

$$w = (1-\alpha_E)p^G(1+s)\frac{e^G}{h^G}$$

$$e = e^Y + c^E$$

$$w^* = (1-\alpha_E)p^{B*}(1-\tau^*)\frac{e^{B*}}{h^{B*}}$$

$$c^* = \frac{w^*}{(h^*)^\phi}$$

$$va^* = (k^{Y*})^\alpha (h^{Y*})^{1-\alpha}$$

$$y = \omega_C \left(\frac{p_H}{p^X}\right)^{-\eta_C} c^X + \omega_I \left(\frac{p_H}{p^I}\right)^{-\eta_I} i + (1-\omega_C)\left(\frac{p_H^*}{p^{X*}}\right)^{-\eta_C} c^{X*} + (1-\omega_I)\left(\frac{p_H^*}{p^{I*}}\right)^{-\eta_I} i^* + g$$

$$y^* = \omega_C \left(\frac{p_F^*}{p^{X*}}\right)^{-\eta_C} c^{X*} + \omega_I \left(\frac{p_F^*}{p^{I*}}\right)^{-\eta_I} i^* + (1-\omega_C)\left(\frac{p_F}{p^X}\right)^{-\eta_C} c^X + (1-\omega_I)\left(\frac{p_F}{p^I}\right)^{-\eta_I} i + g^*$$

B Additional figures

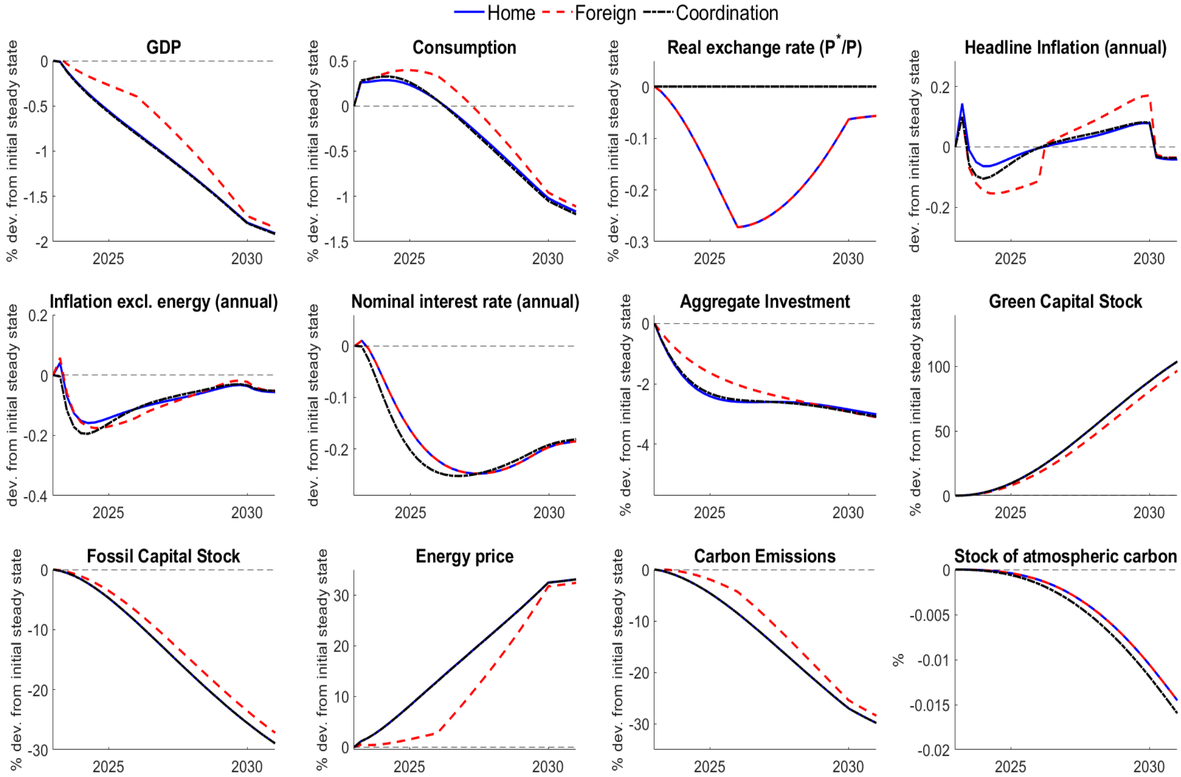

Figure 6: Delayed transition in foreign country (with climate externality)

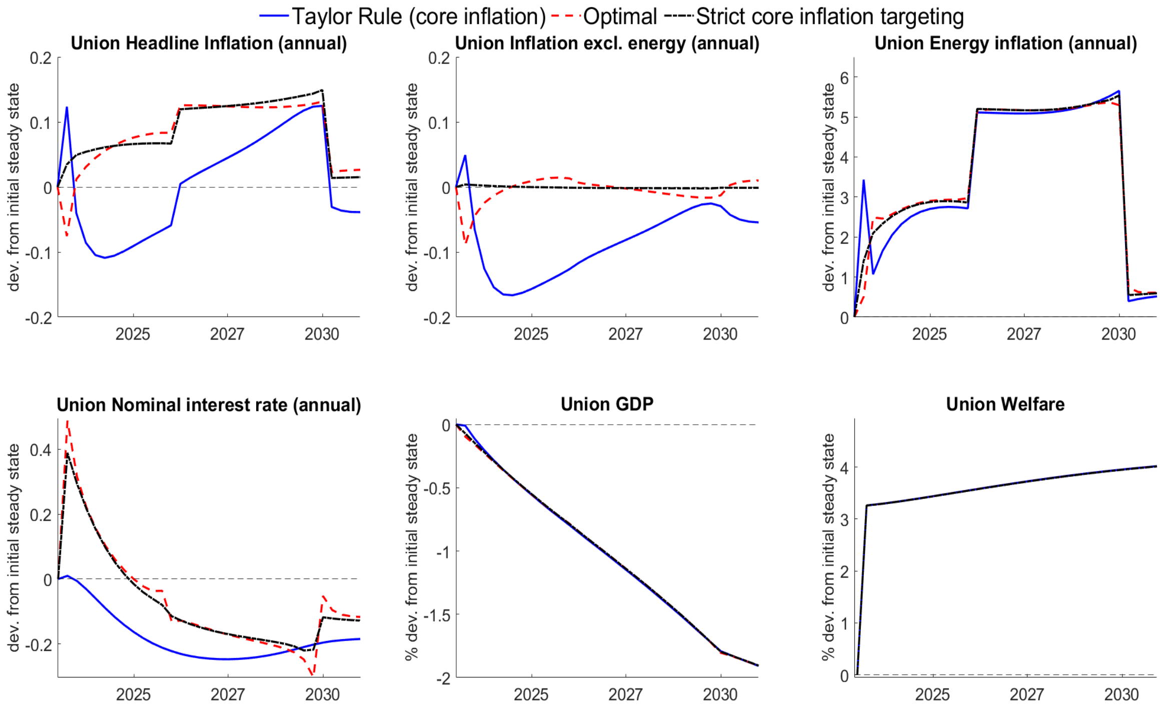

Figure 7: Optimal monetary policy in an uncoordinated transition scenario (with climate externality)